Homework 13

P11.1.15 The switch in Figure P11.1.15 is moved to position ‘b’ at t = 0 after being in position ‘a’ for a long time. Determine,

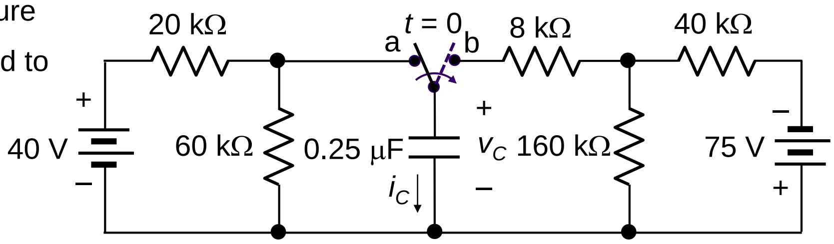

Figure P11.1.15

for $t \geq 0^+$: (a) $v_c(t)$ (b) $i_c(t)$ from initial and final values as well as from the *v-i* relation for the capacitor.

Solution: (a) $v_C(0^-)$ = 40×60/80 = 30 V; after the switch is moved to position ‘b’, $v_C(\infty)$ = -75×160/200 = -60 V; the resistance seen by C is:

8 + 160||40 = 8 + 32 = 40 kΩ; τ_C = 40×10^{-3}×0.25×10^{-6} = 10×10^{-3}; 1/τ_C = 100/s;

$v_C(t)$ = -60 + (30 + 60)e^{-100t} = -60 + 90e^{-100t} V, t is in s.

(b) The circuit at $t = 0^+$ is a two-essential-node circuit. From KCL, $\frac{V_a - 30}{8} + \frac{V_a}{160} + \frac{V_a + 75}{40} =$ 0; this gives V_a = 12 V, $i_C(0^-)$ = (12 – 30)/8 = -2.25 mA. $i_C(\infty)$ = 0, so that

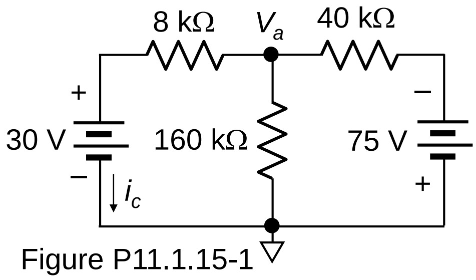

Figure P11.1.15-1

$i_C(t) = -2{,}25e^{-100t}$ mA. From the *v-i*

relation for the capacitor $i_C(t) = C\frac{dv_C}{dt} = 0.25\times10^{-6}\left(-9000e^{-100t}\right) = -2.25e^{-100t}$ mA.

P11.1.23 The switch in Figure P11.1.23 is moved to position ‘b’ at t = 0, after being in position ‘a’ for a long time. Determine, for $t \geq 0^+$, (a) $v_L(t)$; (b) $i_L(t)$. A make-before-break switch is used to avoid open-circuiting the source.

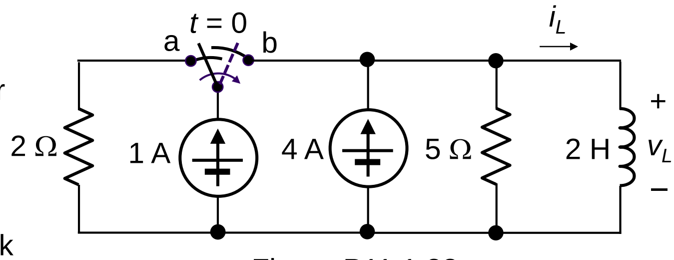

Figure P11.1.23

Solution: (a) Just before the switch is moved, I_{L0}= 4 A. Just after the switch is moved, i_L remains the same and the 1 A source current flows through the 5 Ω resistance, so V_{L0} = 5 V; V_{LF} = 0, and the time constant is 2/5 = 0.4 s. Hence,

v_L = 0 + (5 – 0)$e^{-2.5t}$ = 5$e^{-2.5t}$ V.

(b) I_{L0}= 4 A and I_{LF} = 5 A. Hence, $i_L = 5-(4-5)e^{-2.5t} = 5-e^{-2.5t}$ A.

P11.1.27 The switch in Figure P11.1.27 is opened at t = 0 after being closed for a long time. Determine $i_L(t)$ for $t \geq 0^+$.

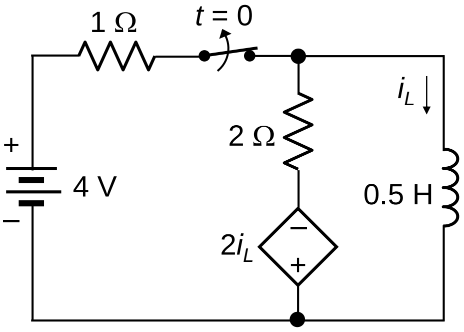

Figure P11.1.27

Solution: After the switch has been closed for a long time, the inductor behaves as a short circuit, so that the voltage across it is zero. This means that there will be a current i_L flowing downwards through the 2 Ω resistor. KCL at the upper node gives 4 = $2I_{L0}$, so that I_{L0} = 2 A. The final value of i_L is zero. When the inductor is replaced by a test source, the circuit becomes as shown. The dependent source is equivalent to a 2 Ω resistor. It follows that V_T/I_T = 4 Ω and τ = 0.5/4 = 1/8 s. Hence, $i_L(t) = 2e^{-8t}$ V, t is in s.

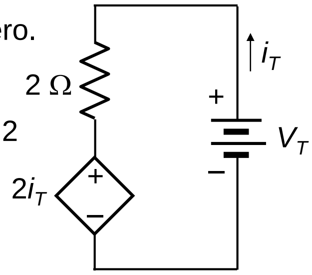

Figure P11.1.27-1

P11.1.29 The switch in Figure P11.1.29, is closed at $t=0$ after being open for a long time. Determine $v_O(t)$ for $t \geq 0^+$.

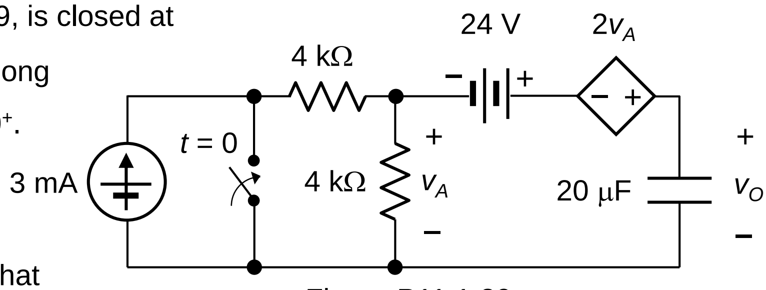

Figure P11.1.29

Solution: When the switch has been open for a long time, the capacitor current is zero, so that $V_A = 4\times3 = 12$ V, and $v_O(0^-) = 3v_A + 24 = 60$ V. As $t \rightarrow \infty$, $V_A = 0$ and $V_{OF} = 24$ V. To find the effective resistance across C, a test source V_T is applied in place of C, as shown. It is seen that $V_T = 3V_A$, where $V_A = 2I_T$; hence, $\frac{V_T}{I_T} = 6$ kΩ, and $\tau = 6\times20 = 120$ ms. It follows that $v_O(t) = 24 + 48e^{-t/0.12}$ V, where t is in s.

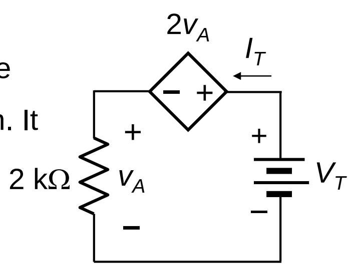

Figure P11.1.29-1

P11.2.5 The switch in Figure P11.2.5 is moved to position 'b' at t = 0 after having been in position 'a' for a long time. Determine $v_O(t)$ for $t \geq 0^+$.

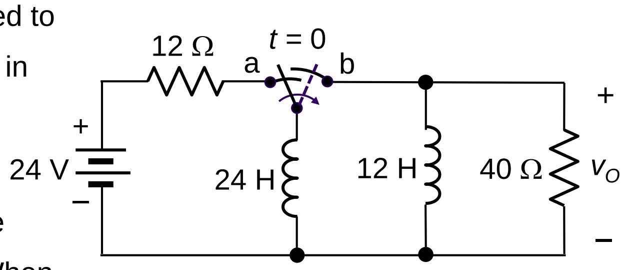

Figure P11.2.5

Solution: When the switch is in position 'a', the current in the 24 H inductor is 2 A. When the switch is moved to position 'b', this current initially flows in the 40 Ω resistor, producing an initial value of v_O that is -2×40 = -80 V. The final value of v_O is zero. The effective inductance is $\frac{12 \times 24}{36} = 8\,\text{H}$. The time constant is $\frac{8}{40} = \frac{1}{5}\,\Omega$. It follows that: $v_O(t) = -80e^{-5t}$ V.

P11.2.9 RC = 1 ms in Figure P11.2.9. Before the switch is closed, the circuit is in a steady state, with a total energy storage of 1 J. If the switch is closed at t = 0, determine the total energy stored at t = 1 ms.

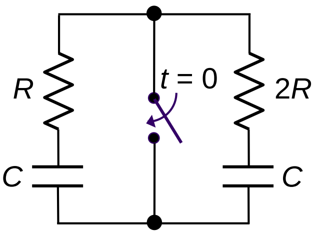

Figure P11.2.9

Solution: When the switch is open, the capacitor voltages are equal, each capacitor storing 0.5 J. If the voltage across each capacitor is V_{c0}, then $0.5CV_{C0}^2$ = 0.5 J.

After the switched is closed, the voltage across the capacitor on the LHS is $V_{C0}e^{-t/RC}$ and the voltage across the capacitor on the RHS is $V_{C0}e^{-t/2RC}$. At t = 1 ms, these voltages are $V_{C0}e^{-1}$ and $V_{C0}e^{-1/2}$. The stored energy is $0.5CV_{C0}^2e^{-2}$ + $0.5CV_{C0}^2e^{-1}$. Substituting $0.5CV_{C0}^2$ = 0.5, the total energy is $0.5(e^{-2} + e^{-1})$ = 0.25 J.

P11.2.16 The switch in Figure P11.2.16 is moved at $t = 0$ from position 'a' to position 'b', after being in position 'a' for a long time. Determine for $t \geq 0^+$: (a) $i_O(t)$; (b) $v_O(t)$.

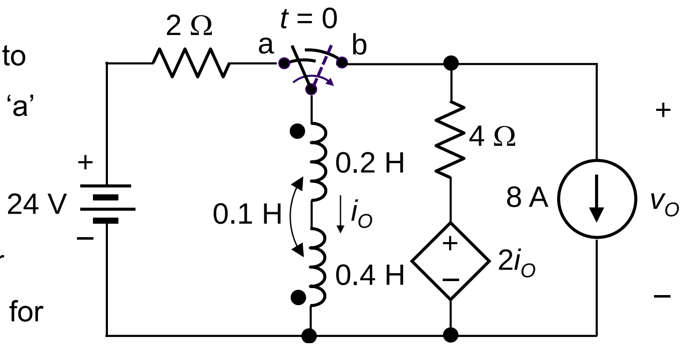

Figure P11.2.16

Solution: $L_{eq} = 0.2 + 0.4 - 0.2 = 0.4$ H. After the switch has been in position 'a' for a long time, the inductors behave as a short circuit, so that $I_O = 12$ A. The circuit for $t \geq 0^+$ is as shown, where $I_{O0} = 12$ A; *so* that $V_{O0} = 24 - 4(12 + 8) = -56$ V. In the steady state, $V_{OF} = 0$, and $2I_{OF} - 4(I_{OF} + 8) = 0$, so that $I_{OF} = -16$ A.

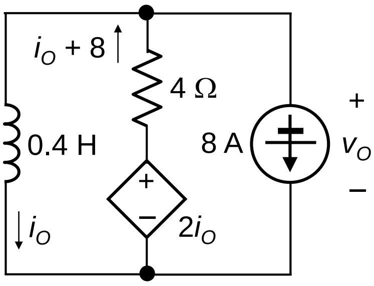

Figure P11.2.16-1

The resistance seen by the inductor is obtained by applying a test source, with the independent source set to zero. $V_T + 2I_T = 4I_T$, which gives $R_{eff} = 2\ \Omega$. It follows that: $i_O(t) = -16 + (12 + 16)e^{-5t} = -16 + 28e^{-5t}$ A, and $v_O(t) = -56e^{-5t}$ V. As a check, $L di_O/dt = 0.4 \times (28)(-5) e^{-5t} = -56e^{-5t}$.

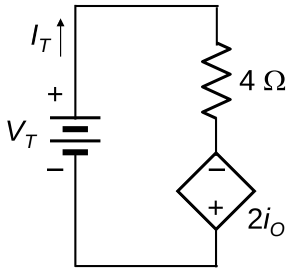

Figure P11.2.16-2

P11.2.17 The switch in Figure P11.2.17 is opened at $t = 0$ with no energy initially stored in the circuit. Determine $v_S(t)$, $t \geq 0^+$.

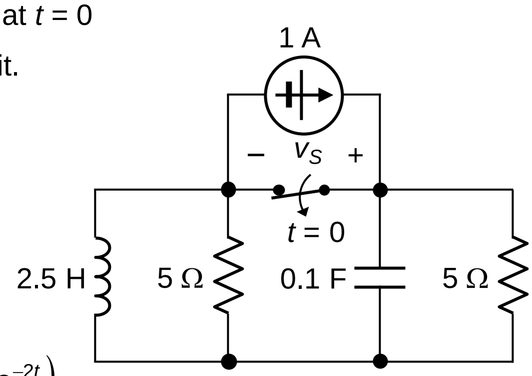

Figure P11.2.17

Solution: After the switch is opened, the current source partitions the circuit into two; $v_C(t)$ is that of an RC parallel circuit charged by a 1 A source. $v_C(0^+) = 0$, $V_{CF} = 5$ V, and $\tau = 0.5$ s. It follows that $v_C(t) = 5\left(1 - e^{-2t}\right)$ V;

$v_L(t)$ is that of an RL parallel circuit charged by a 1 A source. $v_L(0^+) = 5$ V, $V_{LF} = 0$, and $\tau = 2.5/5 = 0.5$ s. It follows that $v_L(t) = 5e^{-2t}$ V; hence,

$v_S(t) = v_C(t) + v_L(t) = 5$ V.

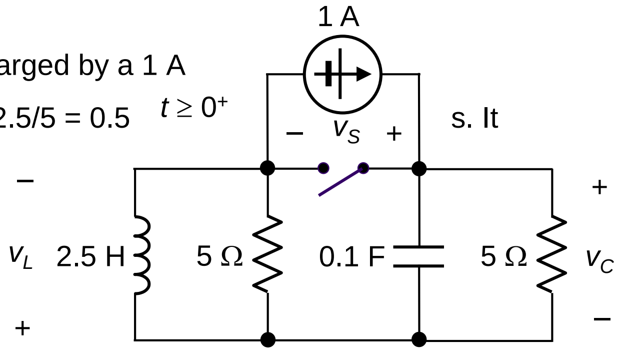

Figure P11.2.17-1